Non Parallels Electric and Magnetic Fields in a F.R.W. Cosmology. Classical and Quantum Gravitational Implications

by

Carlos Pinheiro[1*], *J.A. Helaÿel-Neto*[**] *and Gilmar S. Dias*[***]

[*]Universidade Federal do Espírito Santo, UFES.

Centro de Ciências Exatas

Av. Fernando Ferrari s/nº

Campus da Goiabeiras 29060-900 Vitória ES – Brazil.

[**]Centro Brasileiro de Pesquisas – CBPF/CNPq

Rua Dr. Xavier Sigaud, 150

22290-180 - Rio de Janeiro, RJ – Brasil

Universidade Católica de Petrópolis – UCP

[***]Departamento de Física – UFES

Escola Técnica Federal, ETFES

Abstract

At first, we discuss parallels electric and magnetic fields solutions in a gravitational background. Then, considering eletromagnetic and gravitational waves symmetries we show a particular solution for stationary gravitational waves. Finally we consider gravitation as a gauge theory (effective gravitational theory), evaluate the propagators of the model, analyze the corresponding quantum excitations and verify (confirm) the tree-level unitarity – at many places of the model.

[*]fcpnunes@cce.ufes.br/maria@gbl.com.br

1 Introduction

The physical and mathematical aspects of the vector equations $\vec{\nabla}\times\vec{V}=k\vec{V}$($\vec{V}$ is a vector field and k is a positive constant) have frequently been analyzed. Particularly, plasma and astrophysical plasma physics has pointed out the existence of stationary eletromagnetic waves that seems to be the most important aspect of this equation. Particular solutions have been found for the classical electrodynamics vector potential with the form [1-8]

$$\vec{A}=a[i\ \sin kz+j\cos kz]cos\omega t \tag{1.1}$$

The associated electric and magnetic fields are:

$$\begin{aligned}\vec{E} &= -\frac{1}{c}\frac{\partial\vec{A}}{\partial t}=ka[i\sin kz+\hat{j}\cos kz]\sin\omega t &(1.2)\\ \vec{B} &= \vec{\nabla}\times\vec{A}=ka[i\sin kz+j\cos kz]\cos\omega t &(1.3)\end{aligned}$$

The fields (2) and (3) satisfy Maxwell equations, as well as, the usual vacuum free-wave equation. Moreover, fields $\vec{E}$, $\vec{B}$ and $\vec{A}$ are parallel everywhere. It is interesting to point out, these electromagnetic waves do not propagate energy, having a null Poynting vector. From the $\vec{\nabla}\times\vec{A}=k\vec{A}$, we get the equation:

$$\nabla^2\vec{A}+k^2\vec{A}=0 \tag{1.4}$$

with equation (1) as a possible solution.

We argue that if astrophysical plasma is considered, there must be some restrictions to get the parallel $\vec{A}$, $\vec{E}$ and $\vec{B}$ fields solutions reported by Brownstein et. al. [3, 8].

This work shows that it is not always possible to get these solutions if a strong gravitational background is considered. Gravitation breaks the fields parallelism. So the associated eletromagnetic wave does not have a null Poynting vector anymore and propagates energy in a gravitational background. Then, looking for equation like (1.4) for

gravitational field and a particular solution, we analyze of a possible stationary gravitational wave. It could be new gravitational waves, different from the general relativity gravitational waves (taken from linearization of Einstein equations). Experimentally, this stationary gravitational waves may be found in black-hole distributions.

Finally, using electromagnetic and gravitational gauge theory symmetries we analyse stationary quantum gravitational wave. Here we consider gravitation as a 4-dimensional effective theory.

2 Parallel $\vec{A}$, $\vec{E}$ and $\vec{B}$ fields in Plasma and Astrophysical Plasma

The fields $\vec{A}$, $\vec{E}$ and $\vec{B}$ are parallel in vaccum or, under some conditions, in plamas, what should follow if a gravitational background is taken into account? Do Brownstein fields remain parallel? and does the associated electromagnetic wave propagate energy?

In order to check these questions we consider a gravitational and electromagnetic coupling by the action:

$$\mathcal{S} = \int \sqrt{-g} \left(- \frac{1}{4} \, F_{\mu\nu} F^{\mu\nu} \right) d^4x \tag{2.5}$$

In this gravitational background the Maxwell inhomogeneous equations are:

$$\mathcal{D}_\mu F^{\mu\nu} = \mathcal{J}^\nu \tag{2.6}$$

where

$$\mathcal{D}_\mu F^{\mu\nu} = \partial_\mu F^{\mu\nu} + \Gamma^\mu_{\mu\lambda} F^{\lambda\nu} + \Gamma^\nu_{\mu\lambda} F^{\mu\lambda} = \mathcal{J}^\nu \tag{2.7}$$

and Maxwell homogeneous equations are:

$$\mathcal{D}_\mu F_{\nu\rho} + \mathcal{D}_\nu F_{\rho\mu} + \mathcal{D}_\rho F_{\mu\nu} = 0 \tag{2.8}$$

The connections terms cancel each other such that this last equation is the usual Maxwell homogeneous equation:

$$\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0 \ . \tag{2.9}$$

To solve inhomogeneous equations (8) we adopt the F.R.W. cosmological metric:

$$dS^2 = dt^2 - a^2_{(t)} \left\{ \left(1 - Ar^2\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \right\} \tag{2.10}$$

At the same time, the second term of the covariant derivative equation (7) may be written appropriately, with $\Gamma^\mu_{\mu\lambda}$ given as

$$\Gamma^\mu_{\mu\lambda} = -\ \frac{\partial}{\partial x^\lambda}\ \log\sqrt{-\tilde{g}}\ , \tag{2.11}$$

where $\tilde{g}$ is the metric determinant.

Then Maxwell equations (7) and (8) are explicitly given by:

$$\vec{\nabla}\cdot\vec{E} - g\vec{\nabla}f\cdot\vec{E} = \rho_{(\vec{x})}\ , \tag{2.12}$$

$$\vec{\nabla}\cdot\vec{B} = 0\ ,$$

$$\vec{\nabla}\times\vec{E} = -\ \frac{\partial\vec{B}}{\partial t}\ ,$$

$$\vec{\nabla}\times\vec{B} = \vec{J}(\vec{x}) + \frac{\partial\vec{E}}{\partial t} - g\ \frac{\partial f}{\partial t}\ \vec{E} + g\vec{\nabla}f\times\vec{B} - \Gamma^i_{\mu\lambda}F^{\mu\lambda}\ .$$

Without electromagnetic sources, that is, for $\rho = 0$ and $\vec{J} = 0$ we get "free" electromagnetic fields equations in a graviational background:

$$\vec{\nabla}\cdot\vec{E} = g\vec{\nabla}f\cdot\vec{E}\ , \tag{2.13}$$

$$\vec{\nabla}\cdot\vec{B} = 0\ , \tag{2.14}$$

$$\vec{\nabla}\times\vec{E} = -\ \frac{\partial\vec{B}}{\partial t}\ , \tag{2.15}$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} - g\ \frac{\partial f}{\partial t} \vec{E} + g \vec{\nabla} f \times \vec{B} - \Gamma^{i}_{\mu\lambda} F^{\mu\lambda} \tag{2.16}$$

where. $g = \dfrac{\sqrt{1 - Ar^2}}{a^3 r^2 \sin\theta}$ and $f = \dfrac{a^2 r^2 \sin\theta}{\sqrt{1 - Ar^2}}$.

Functions g and f may have $A = +1, 0, -1$ each value represents the associated curvature of F.R.W. spatial metric section.

The electric field wave equation in a gravitational background is obtained from eq. (15) as

$$\begin{aligned} \nabla^2 \vec{E} \quad - \quad \frac{\partial^2 \vec{E}}{\partial t^2} &= \vec{\nabla} \left(g \vec{\nabla} f \cdot \vec{E} \right) - \frac{\partial g}{\partial t}\ \frac{\partial f}{\partial t}\ \vec{E} - g\ \frac{\partial^2 f}{\partial t^2}\ \vec{E} - g\ \frac{\partial f}{\partial t}\ \frac{\partial \vec{E}}{\partial t} + \\ + \quad \frac{\partial g}{\partial t}\ \vec{\nabla} f \times \vec{B} &+ g\ \frac{\partial}{\partial t}\ \vec{\nabla} f \times \vec{B} + g \vec{\nabla} f \times \frac{\partial \vec{B}}{\partial t} - \frac{\partial}{\partial t}\ \left(\Gamma^{i}_{\mu\beta} F^{\mu\beta} \right)\ . \end{aligned} \tag{2.17}$$

The magnetic field wave equation in a gravitational background is obtained from eq. (16) as

$$\nabla^2 \vec{B} - \frac{\partial^2 \vec{B}}{\partial t^2} = \vec{\nabla} \times \left(g\ \frac{\partial f}{\partial t}\ \vec{E} - g \vec{\nabla} f \times \vec{B} + \Gamma^{i}_{\mu\beta} F^{\mu\beta} \right)\ . \tag{2.18}$$

The last term, $\Gamma^{\nu}_{\mu\lambda} F^{\mu\lambda}$, is identically zero. But we still retain it for esthetic completeness of equation (2.7). At the same time, equations (2.17) and (2.18) reproduce the correponding vacuum equations if gravitation is ignored.

Using equation (2.17) and (2.18) it can be shown that there is at least, one solution in which fields $\vec{E}$ and $\vec{B}$ are not parallel anymore. For example, putting field $\vec{B}$ as given by (2.3), in equation (2.18), and writting the gradient $\vec{\nabla} f$ as:

$$\vec{\nabla} f = \hat{r}_0 \frac{\partial f}{\partial r} + \hat{\theta}_0 \frac{\partial f}{\partial \theta} + \hat{\varphi}_0\ \frac{\partial f}{\partial \varphi}\ , \tag{2.19}$$

it is easy to verify that equation (2.18) becomes

$$\frac{\partial f}{\partial t}\ \vec{E} = \vec{\nabla} f \times \vec{B} \Longrightarrow \vec{E} = \frac{\vec{\nabla} f \times \vec{B}}{\dfrac{\partial f}{\partial t}} \tag{2.20}$$

Calculating $\dfrac{\partial f}{\partial t}$, $\vec{\nabla} f$, $\vec{\nabla} f \times \vec{B}$ and taking explicitly the unity vectors $\hat{r}_0$ and $\hat{\theta}_0$

$$\begin{aligned} \hat{r}_0 &= \hat{\imath} \sin\theta \cos\varphi + \hat{\jmath} \sin\theta \sin\varphi + \hat{k} \cos\theta \\ \hat{\theta}_0 &= \hat{\imath} \cos\theta \cos\varphi + \hat{\jmath} \cos\theta \sin\varphi - \hat{k} \sin\theta \end{aligned} \tag{2.21}$$

we can find,

$$\hat{B} = ka \left[i \sin(kz) + \hat{\jmath}(kz) \right] \cos(\omega t) \ , \tag{2.22}$$

$$\begin{aligned} \vec{E} &= \hat{\imath} \left(\sin\theta G_{(r,t,\theta)} - \cos\theta F_{(r,t)} \right) ka \cos(kz) \cos(\omega t) + \\ &+ \hat{\jmath} \left(\cos\theta F_{(r,t)} - \sin\theta G_{(r,t,\theta)} \right) ka \sin(kz) \cos(\omega t) + \\ &+ \hat{k} \left[\left(\sin\theta \cos\varphi F_{(r,t)} + \cos\theta \cos\varphi G_{(r,t,\theta)} \right) ka \cos(kz) \cos(\omega t) + \right. \\ &- \left. \left(\sin\theta \sin\varphi F_{(r,t)} \cos\theta \sin\varphi G_{(r,t,\phi)} \right) ka \sin(kz) \cos(\omega t) \right] \end{aligned} \tag{2.23}$$

where functions $F_{(r,t)}$ and $G_{r,t,\theta)}$ are given by:

$$\begin{aligned} F_{(r,t)} &= \frac{2a}{3\dot{a}r} + \frac{Aar}{3\dot{a}(1 - Ar^2)} \quad \text{and} \\ G_{(r,t,\theta)} &= \frac{a\mathrm{cotg}\theta}{3\dot{a}r} \ . \end{aligned}$$

The new electrital field $\vec{E}$ given by (2.23) is completely different from the Brownstein electrical field (1.2). Now, fields $\vec{E}$ and $\vec{B}$ are perpendicular vectors, as we can see from equation (2.20). Gravitational backgound breaks $\vec{E}$ and $\vec{B}$ parallelism and so the Poynting vector $\vec{S} = \vec{E} \times \vec{B}$ is not zero anymore.

This way, the associated electromagnetic wave propagates energy.

It is possible the electric field as the Brownstein electric field given in eq. (1.2) is kept and a new particular solution for magnetic $\vec{B}$ field in a gravitational background is obtained. We believe, this new magnetic field differs from the Brownstein magnetic field given in eq. (1.3) and the associated electromagnetic wave propagates energy.

It is not always possible to have $\vec{E}$, $\vec{B}$ and $\vec{A}$ as parallel fields if a gravitational background is taken into account. On the other hand, even in a gravitational background, it might be possible to claim parallel $\vec{E}$ and $\vec{B}$ fields, writing $\vec{E} = \chi\vec{B}$ in equations (2.17) and (2.18) and to get conditions that the function χ must satisfy.

3 Stationary Gravitational Waves

Now, considering the similarities of electromagnetic, gravitational and linearized Einstein gravitation theories, we analyze a possibility of stationary gravitational wave.

According to the phenomenological view, black-hole distributions may be nodes of gravitational waves propagating in space. It would be sufficient for two black-holes separated by large distances, if the space-time curvature of the strong gravitational fields of the black-holes is neglected. So the space-time of one black-hole is assymptotically flat to the other. Then each black-holes is like a 'string node' and this mechanism may confine gravitational waves.

The following equation may reproduce these waves in nature:

$$R_{\mu\nu} = \kappa\Lambda h_{\mu\nu} \tag{3.1}$$

where

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \ . \tag{3.2}$$

Here κ is the Newton gravitational constant, Λ is the cosmological constant, $h_{\mu\nu}$ is the stationary gravitational perturbation in a Minkowski "Background". Then the stationary gravitational pertubation is given by

$$\partial_\beta\partial_\nu h^\beta_\mu + \partial_\beta\partial_\mu h^\beta_\nu - \Box h_{\mu\nu} - \partial_\mu\partial_\nu h^\beta_\beta = \Lambda h_{\mu\nu} \ . \tag{3.3}$$

This equation is formally similar to the electromagnetic case, eq. (1.4).

This equation may have a solution like

$$h_{\mu\nu} = C_{\mu\nu}(z) f(t) \ , \tag{3.4}$$

where

$$h_{\mu\nu} = \begin{bmatrix} A_{00} & 0 & 0 & 0 \\ 0 & A_{11} & A_{12} & 0 \\ 0 & A_{12} & -A_{11} & 0 \\ 0 & 0 & 0 & A_{00} \end{bmatrix} e^{i\tilde{k}z} \cos\omega t \ , \tag{3.5}$$

where A_{00}, A_{11} and A_{12} are free constants and $\tilde{k} = \sqrt{\Lambda - \omega^2}$ is the wave-vector for the case $\Lambda - \omega^2 > 0$.

The dispersion relation suggests a long wave-length, λ, for stationary gravitational waves in nature, since its frequency is small.

The lagrangean for the pertuberd field is

$$\begin{aligned} \mathcal{L}_h^{(\Lambda)} &= -\frac{1}{4}\,\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} + \frac{1}{4}\,\partial_\rho h \partial^\rho h - \frac{1}{2}\,\partial_\rho h^{\rho\mu}\partial_\mu h + \frac{1}{2}\,\partial^\rho h_{\rho\mu}\partial_\nu h^{\mu\nu} + \\ &- \frac{1}{4}\,\Lambda h^{\mu\nu} h_{\mu\nu} + \frac{1}{8}\,\Lambda h^2 \ . \end{aligned} \tag{3.6}$$

¿From this lagrangean we get the energy-momentum tensor $T_{\mu\nu}$ and, using the equation of motion with $\partial_\mu T^{\mu\nu} = 0$. The stress tensor is not symmetric. Then $T_{\mu\nu}$ is

$$\begin{aligned} T_{\mu\nu} &= \frac{1}{4}\,\partial_\mu\partial_\nu h - \frac{1}{2}\,\partial_\mu h_{\alpha\beta}\partial_\nu h^{\alpha\beta} - \frac{1}{2}\,\partial_\nu h_{\mu\beta}\partial^\beta h - \frac{1}{2}\,\partial^\beta h_{\mu\beta}\partial_\nu h + \\ &+ \partial^\beta h_{\beta\sigma}\partial_\nu h_\mu^\sigma - \eta_{\mu\nu}\mathcal{L}_h^{(\Lambda)} \ . \end{aligned} \tag{3.7}$$

A system of black-hole distribution may confine stationary gravitational waves.

4 Gravitation as a Gauge Theory-Gravitons

Now, we intend to attack the last problem according to quantum view. Gravitation is considered a gauge theory.

Classically, stationary gravitational waves between two points x and y have been obtained. According to the quantum view, this problem may have graviton exchanges. The situation is similar to that of point charges interacting by photon exchanges.

This way, the quantum version of the problem of stationary gravitational waves between two points x and y is now described by graviton creation and annihilation at these same points.

For convecinece, we parametrize the field $h_{\mu\nu}$ as

$$H^{\alpha}_{\nu} = h^{\alpha}_{\nu} - \frac{1}{2}\,\delta^{\alpha}_{\nu} h \ .$$

It is simple to verify that $H^{\alpha}_{\alpha} = H = -h$ where h is the trace of $h_{\mu\nu}$. With this new $h_{\mu\nu}$, the Lagrangian (29) may be written as

$$\begin{aligned} \mathcal{L}^{(\Lambda)}_{H} &= \frac{1}{2}\,\partial_{\rho} H_{\mu\nu} \partial^{\rho} H^{\mu\nu} + \frac{1}{4}\,\partial_{\rho} H \partial^{\rho} H - \frac{1}{2}\,\partial^{\rho} H_{\rho\nu} \partial_{\mu} H^{\mu\nu} + \frac{1}{2}\,\partial^{\rho} H_{\rho\mu} \partial_{\nu} H^{\mu\nu} + \\ &- \frac{1}{2}\,\Lambda H^{\mu\nu} H_{\mu\nu} + \frac{1}{4}\,\Lambda H^{2} \ . \end{aligned} \tag{4.8}$$

Finally, we integrate the lagrangean (31) by parts and get:

$$\begin{aligned} \mathcal{L}^{(\Lambda)}_{H} &= \frac{1}{2}\,H^{\mu\nu} \Box H^{\mu\nu} - \frac{1}{4}\,H \Box H - \frac{1}{2}\,H^{\mu\nu} \partial_{\mu} \partial_{\alpha} H^{\alpha}_{\nu} - \frac{1}{2}\,H^{\mu\nu} \partial_{\nu} \partial_{\alpha} H^{\alpha}_{\nu} + \\ &- \frac{1}{2}\,\Lambda H^{\mu\nu} H_{\mu\nu} + \frac{1}{4}\,\Lambda H^{2} \ . \end{aligned} \tag{4.9}$$

Then, it is possible to write this expression in a bilinear form:

$$\mathcal{L}^{(\Lambda)}_{H} = \frac{1}{2}\,H^{\mu\nu} \Theta_{\mu\nu,\kappa,\lambda} H^{\kappa,\lambda} \ , \tag{4.10}$$

where operator $\Theta_{\mu\nu,\kappa\lambda}$ has the following form in terms of Barnes-Rivers [9] spin projection operators:

$$\begin{aligned} \Theta_{\mu\nu,\kappa\lambda} &= (\Box - \Lambda) P^{(2)} - \Lambda P^{(1)}_{m} + \frac{5}{2}\,(\Box - \Lambda) P^{(0)}_{s} - \frac{(\Lambda + 3\Box)}{2}\,P^{(0)}_{\omega} \\ &+ \frac{\sqrt{3}}{2}\,(\Lambda - \Box) P^{(0)}_{\omega} + \frac{\sqrt{3}}{2}\,(\Lambda - \Box) P^{(0)}_{\omega s} \ . \end{aligned} \tag{4.11}$$

The inverse operator $\Theta^{-1}_{\mu\nu,\kappa\lambda}$ has the following form:

$$\Theta^{-1}_{\mu\nu,\kappa\lambda} = \left[XP^{(2)} + YP^{(1)}_m + ZP^{(0)}_s + WP^{(0)}_\omega + RP^{(0)}_{sw} + SP^{(0)}_{\omega s}\right]_{\mu\nu,\kappa\lambda} \ . \tag{4.12}$$

To get the coefficients $X,\ Y\ \cdots S$ it is sufficient to use Barnes-Rivers [9] projection operators and its multiplicative table, such that:

$$\Theta^{\rho\sigma}{}_{\mu\nu}\Theta^{-1}_{\rho\sigma,\kappa\lambda} = (I)_{\mu\nu,\kappa\lambda} = \left(P^{(2)} + P^{(1)}_m + P^{(0)}_s + P^{(0)}_\omega\right)_{\mu\nu,\kappa\lambda} \ . \tag{4.13}$$

For $D = 4$ these coefficients have the following form

$$X = -\ \frac{1}{\Lambda - \Box}\ , \quad Y = -\ \frac{1}{\Lambda}\ , \quad Z = -\ \frac{\Lambda + 3\Box}{\Lambda^2 + 8\Lambda\Box - 9\Box^2}\ , \tag{4.14}$$

$$W = -\ \frac{5}{\Lambda - 9\Box}\ , \quad R = -\ \frac{\sqrt{3}}{\Lambda + \Box}\ , \quad S = -\ \frac{\sqrt{3}}{\Lambda + 9\Box}\ .$$

We get the corresponding propagator by the following functional generator:

$$W[T_{\rho\sigma}] = -\ \frac{1}{2}\int d^4x d^4y T^{\mu\nu}\Theta^{-1}_{\mu\nu,\kappa\lambda}T^{\kappa\lambda}\ .$$

The propagator is written explicitly as:

$$\langle T\left[h_{\mu\nu(x)}; h_{\kappa\lambda(x)}\right]\rangle = i\Theta^{-1}_{\mu\nu,\kappa\lambda}\delta^4(x-y)\ . \tag{4.15}$$

5 Unitarity of the Model

Now we discuss the tree-level unitarity of the model. Coupling the propagator and external current, $T^{\mu\nu}$ we analyze the poles of the current amplitude and the imaginary part of its residue. The current amplitude is given by:

$$\mathcal{A} = T^{*\mu\nu}(\vec{k})\langle T\left[h_{\mu\nu}(-\vec{k}); h_{\kappa\lambda}(\vec{k})\right]\rangle T^{\kappa\lambda}(-\vec{k})\ . \tag{5.16}$$

Using equations (4.12), (4.14) and (4.15) we get the current amplitude. And considering the imaginary part of the residue at the poles, it is simple to verify that, due to the transverse condition, only operators $P^{(2)}$ and $P_s^{(0)}$ remain. And that it is a symmetry, not necessarily a gauge symmetry:

$$\omega_{\mu\nu} T^{\mu\nu} = 0 \ . \tag{5.17}$$

Analyzing the poles of the sector with spin-2 and the sector with spin-0 in the momentum space we get:

$$X = -\ \frac{1}{k^2 + \Lambda} \tag{5.18}$$

$$Z = \frac{3k^2 - \Lambda}{\Lambda^2 - 8\Lambda k^2 - 9(k^2)^2} \ .$$

This way we have for the sector with spin-2, a tachyon pole:

$$k^2 = -\Lambda \tag{5.19}$$

and for the sector with spin-0, we have two poles, one being a tachyon pole

$$\begin{aligned} k^2 &= \frac{1}{9}\,\Lambda \ , \\ k^2 &= -\Lambda \ . \end{aligned} \tag{5.20}$$

Using (5.16) and considering the k^2 pole analysis we can verify the tree-level unitarity of the model. But we want spin-2 gravitons, so we redefine Λ as $(-\Lambda)$ and, so, for the spin-2 sector, we have

$$k^2 = \Lambda \ . \tag{5.21}$$

For the spin-0 sector, we have, respectively:

$$k^2 = -\ \frac{1}{9}\,\Lambda \ , \tag{5.22}$$

$$k^2 = \Lambda \ .$$

Now we have one non tachyonic pole for the spin-2 sector, one non tachyonic and one tachyonic pole for the spin-0 sector.

And we find

$$Im\ Res\mathcal{A}|_{k^2=\Lambda} > 0 \tag{5.23}$$

Thus the massive excitation is, in fact, a dynamical degree of freedom. The theory does not have negative norm states. And this is the unitarity requisite to have the required assymptotic behavior. From (41), it is simple to verify that the propagator is proportional to $\frac{1}{k^4}$, and therefore the $4D$ model is not renormalizable.

So we conclude that the model discussed is causal, has tree-level unitarity and is not renormalizable by power counting. The proposed model has to be understood as an "effective gravitation theory" with a physical massive degree of freedom ($k^2 = \Lambda$).

Acknowledgements:

The authors want to thank G.O. Pires for his comments and technical discussions and S.A. Diniz for his work on typing. We also thank Prof. Berth Schröer for many discussions during his visit to the Department of Physics of UFES. We also would like to thank the Department of Physics, University of Alberta for their hospitality. Finally we thank Conselho Nacional de Pesquisa – CNPq – for the financial support.

References

[1] K.R. Brownstein, Phys. Rev. A 35, 4854 (1987);

[2] N. Salingaros, J. Phys. A 19, L 101 (1986);

[3] G.F. Freire, Am. J. Phys. 34, 567 (1966);

[4] C. Chu and T. Ohkawa, Phys. Rev. Lett. 48, 837 (1982);

[5] K.K. Lee, Phys. Rev. Lett. 50, 138 (1983);

[6] N. Salingaros, Am. J. Phys. 53, 361 (1985);

[7] C. Chu, Phys. Rev. Lett. 50, 139 (1983);

[8] M. Maheswaran, J. Phys. A 19, L761 (1986);

[9] C. Pinheiro and G.O. Pires, Phys. Letters B. 301 339 (1993);

[10] George Arfken, Mathematical Methods for Physicists, second edition (Academic Press).

[11] Carlos Pinheiro, G.O. Pires and Nazira Tomimmura, General Relativity and Gravitation, vol. 29, N$^{\underline{0}}$ 4, 409 (1997).

[12] Carlos Pinheiro, G.O. Pires and C. Sasaki and Il Nuovo Cimento B, vol. 111B, N$^{\underline{0}}$ 8, 1023-1028 (1996).

[13] Carlos Pinheiro, G.O. Pires and F.A.B.R. de Carvalho – Brazilian J. of Phys., vol. 27, 14 (1997).